Efficient Quantum Hermite Transform

Sid Jain

Joint work with Vishnu Iyer, Rolando Somma, Ning Bao, and Stephen Jordan

Quantum algorithms for continuous functions?

arXiv > quant-ph > arXiv:quant-ph/0405146

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Quantum Physics

[Submitted on 25 May 2004 (v1), last revised 2 Jan 2005 (this version, v2)]

Fast quantum algorithm for numerical gradient estimation

Stephen P. Jordan

Given a blackbox for f, a smooth real scalar function of d real variables, one wants to estimate the gradient of f at a given point with n bits of precision. On a classical computer this requires a minimum of d+1 blackbox queries, whereas on a quantum computer it requires only one query regardless of d. The number of bits of precision to which f must be evaluated matches the classical requirement in the limit of large n.

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What about QFT?

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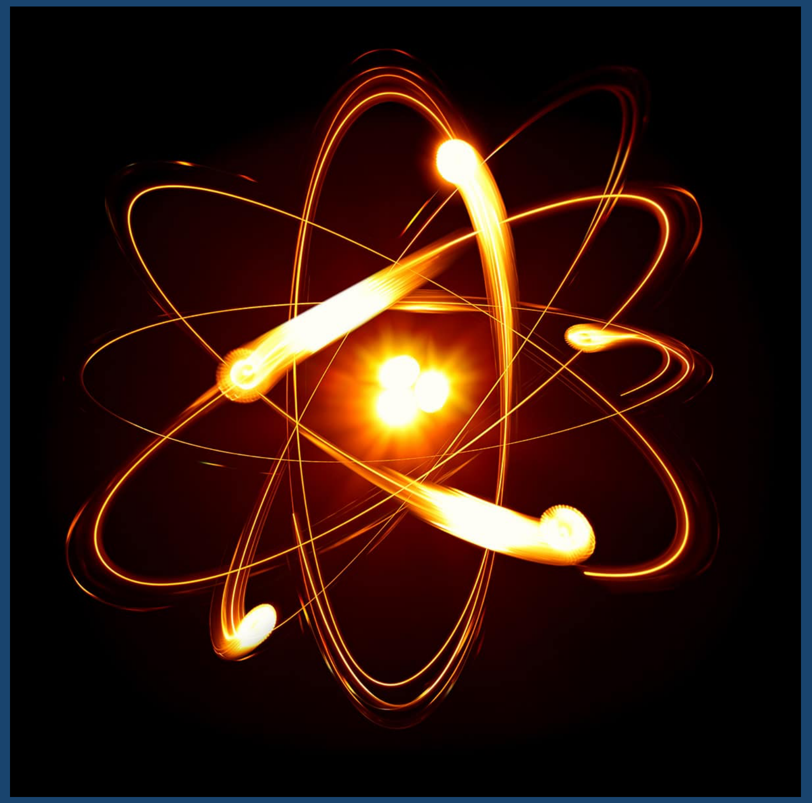

Quantum physics

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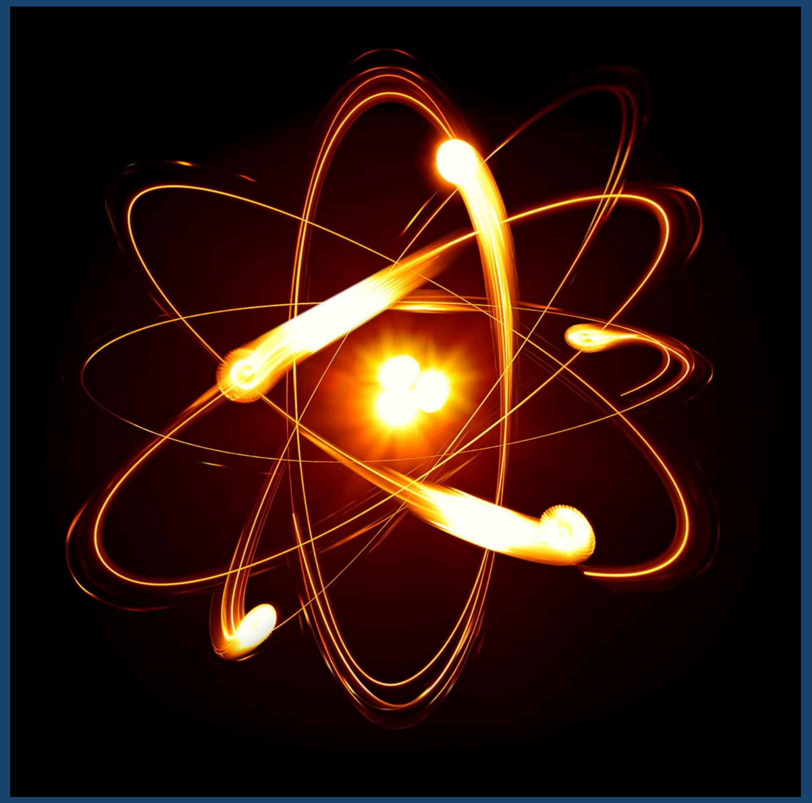
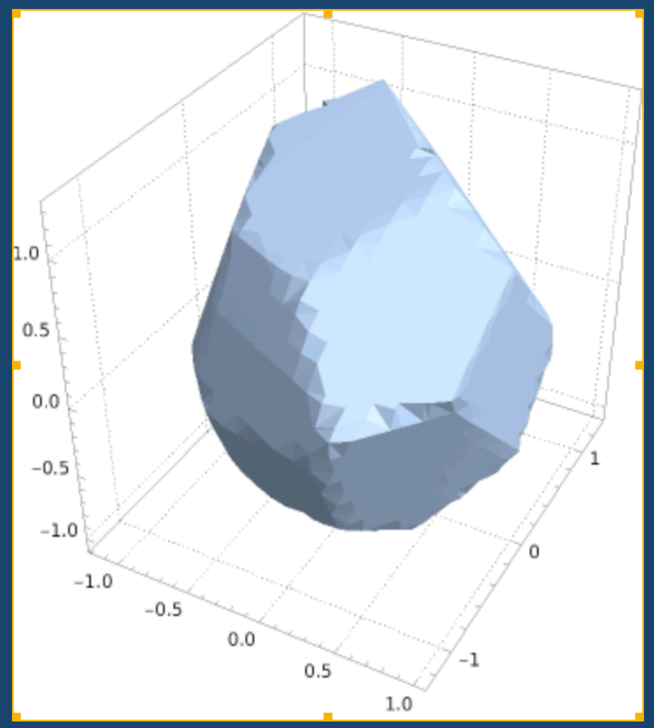


Quantum physics



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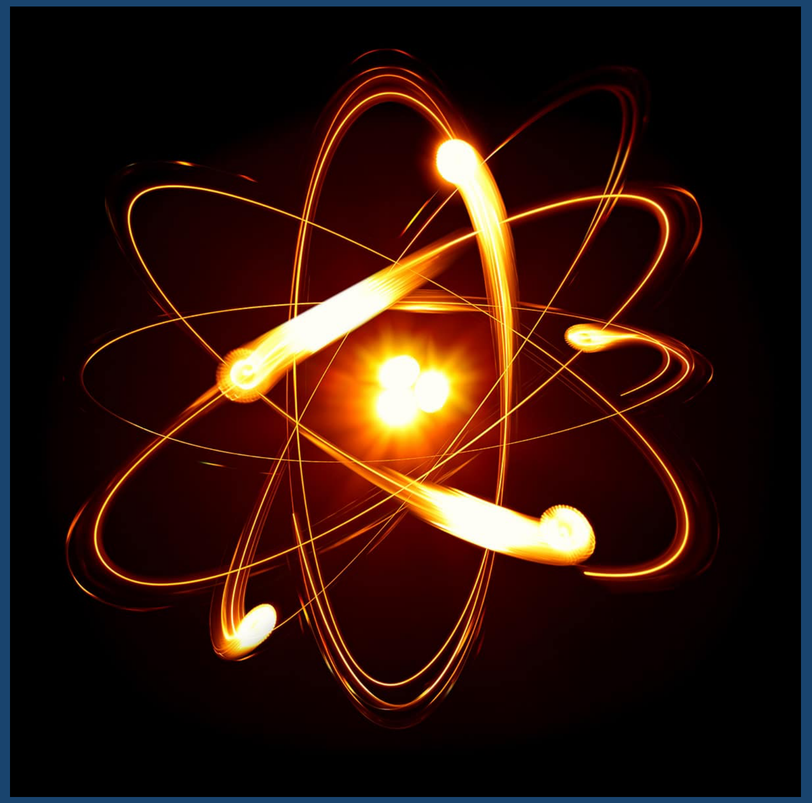

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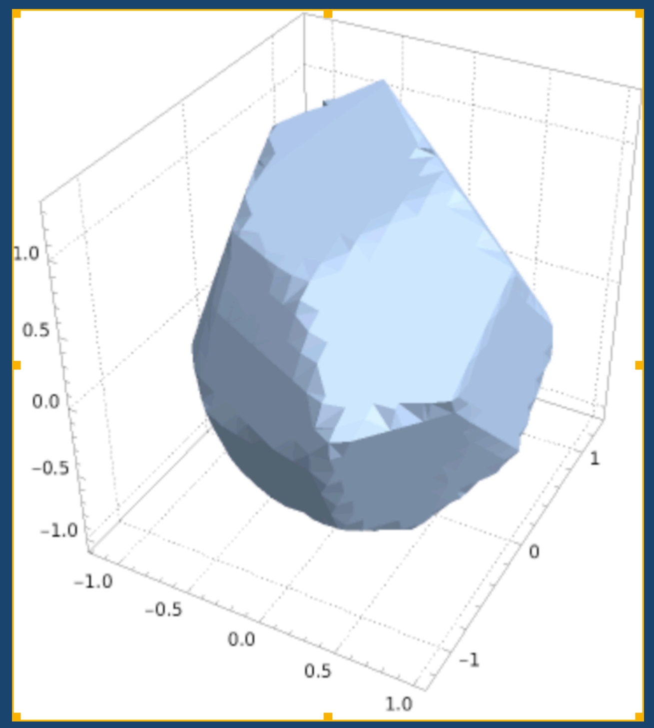

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$$\nabla^2 \cdot u = \frac{\partial u}{\partial t}$$

Differential equations

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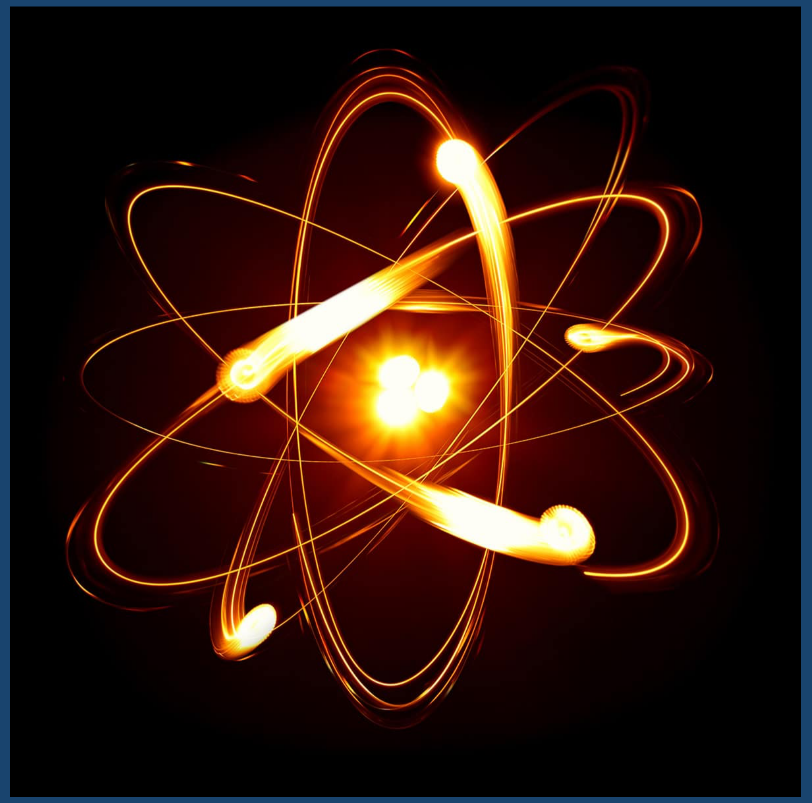

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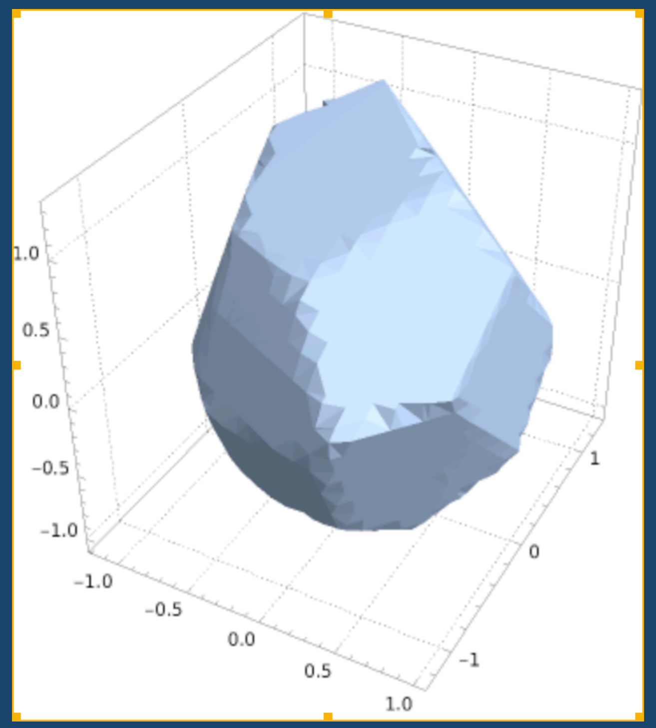

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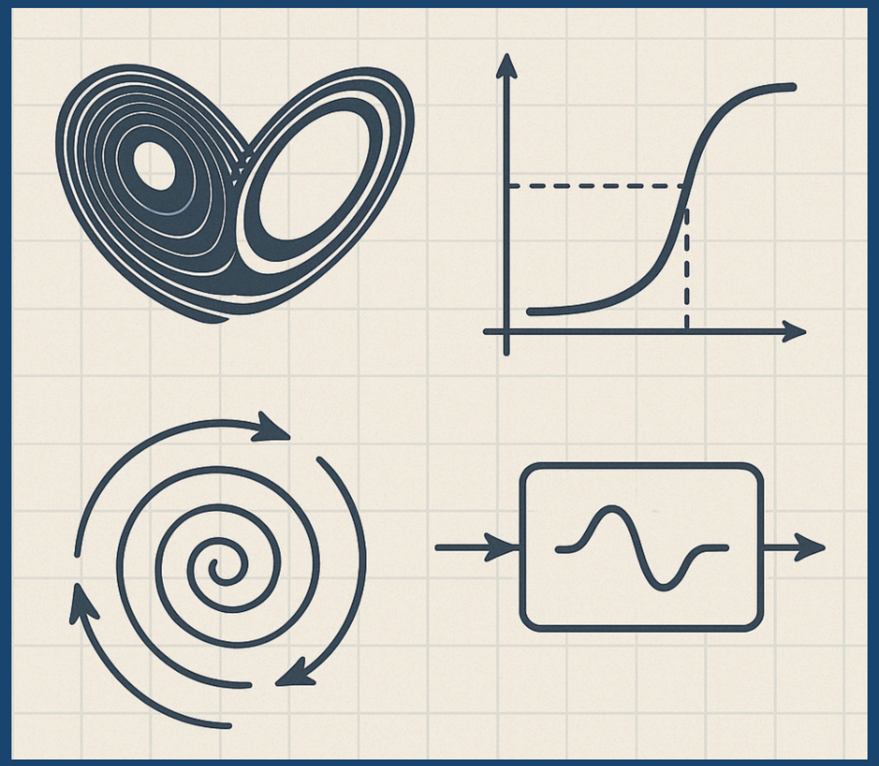

Signals and systems

Quantum harmonic oscillator

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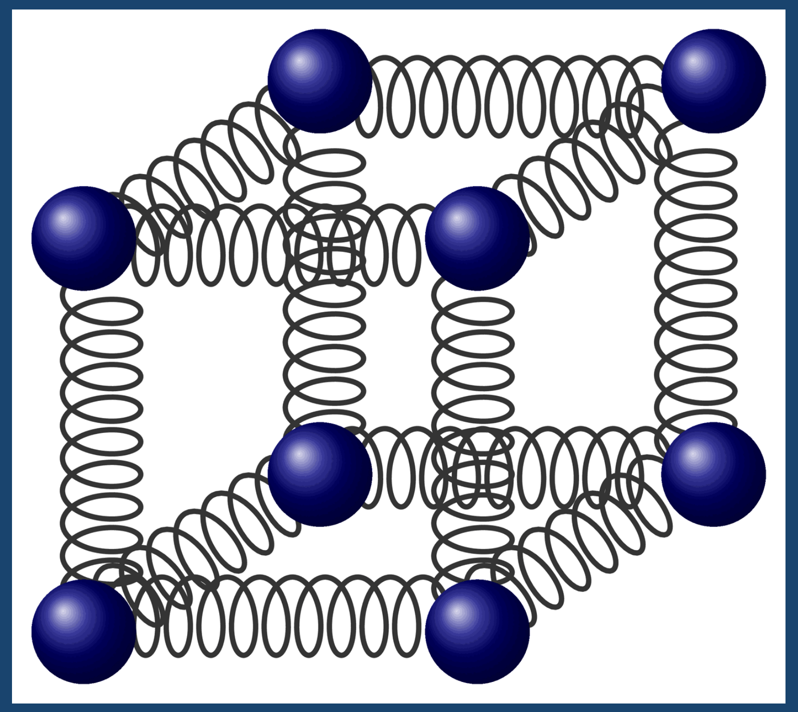

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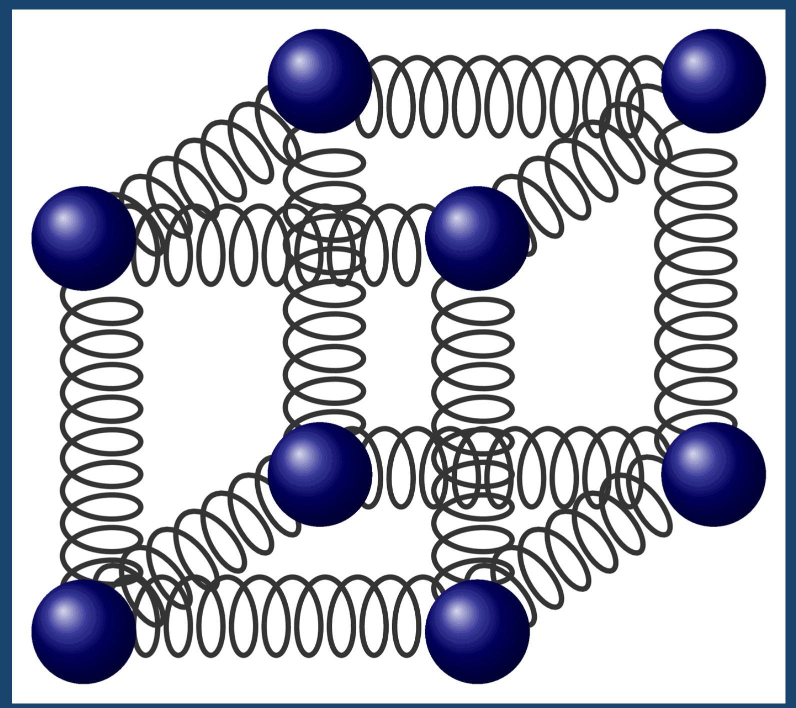

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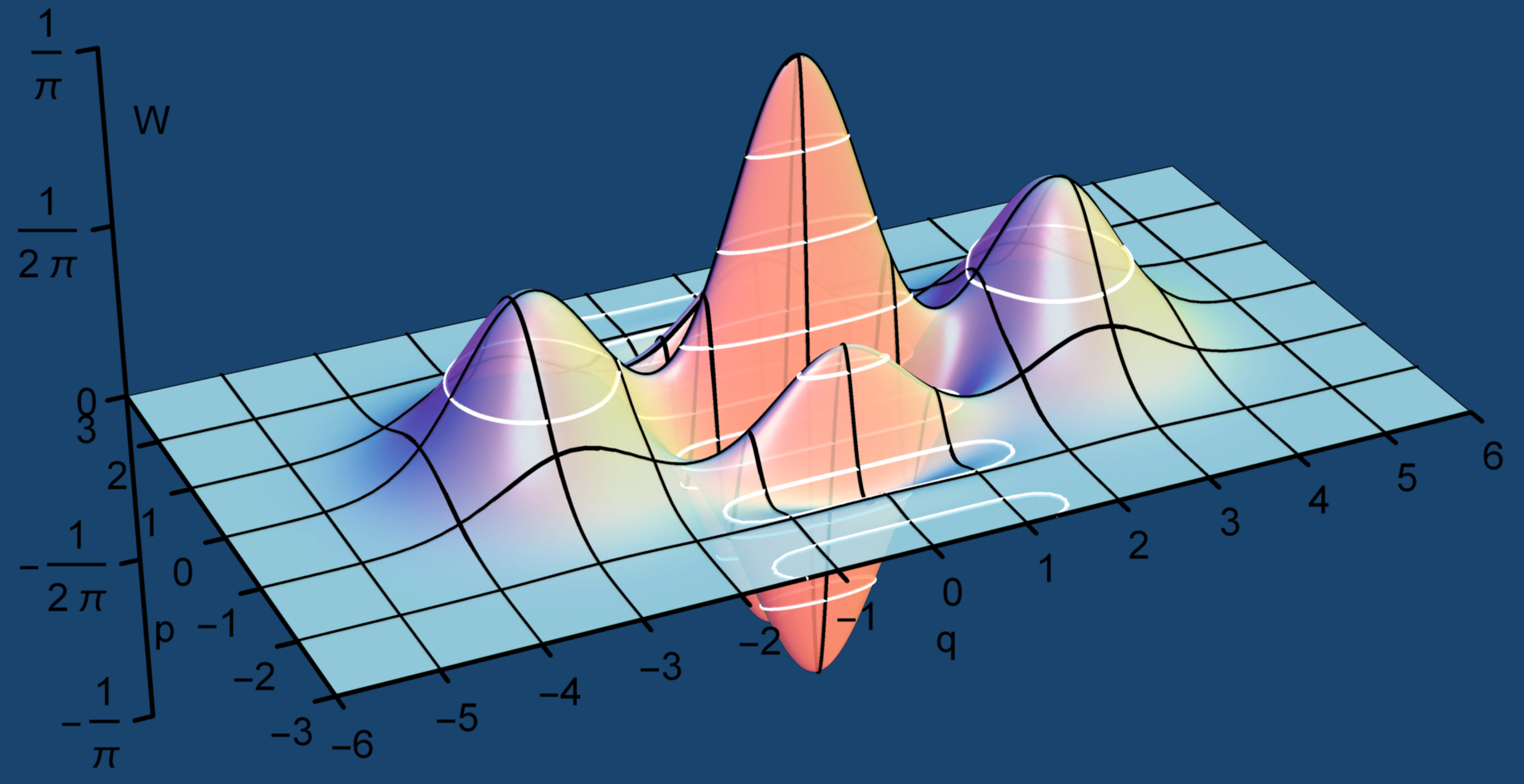

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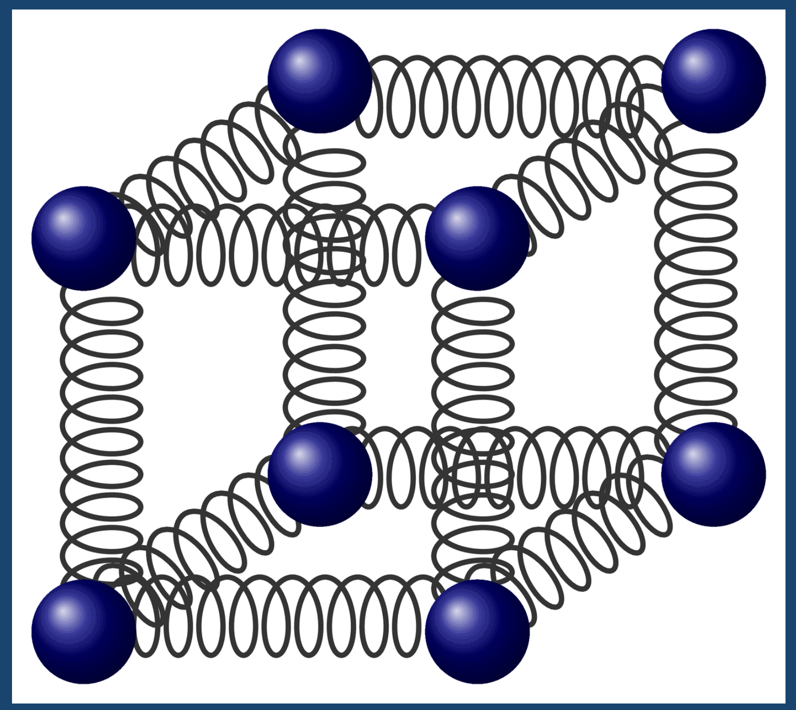

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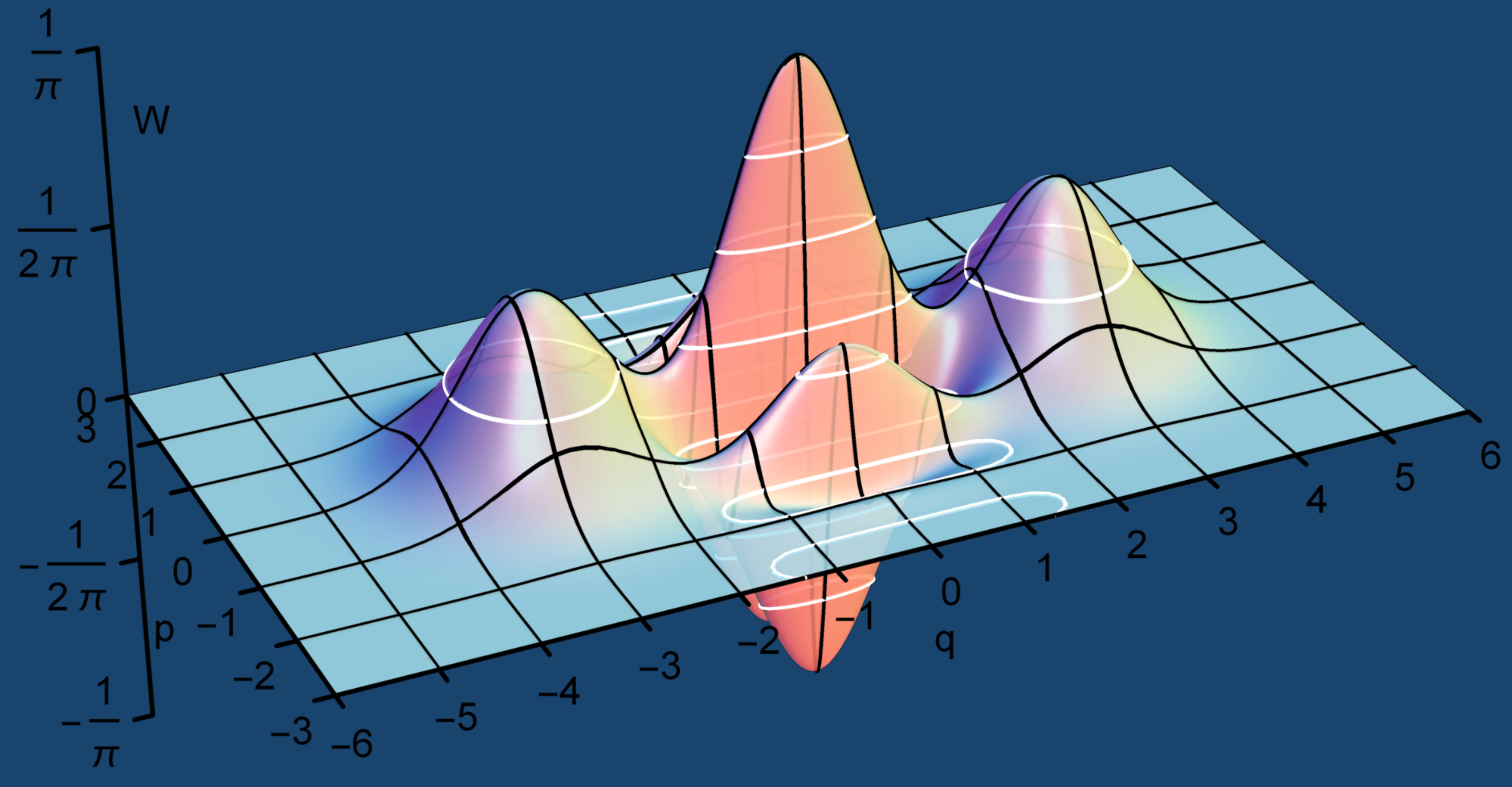

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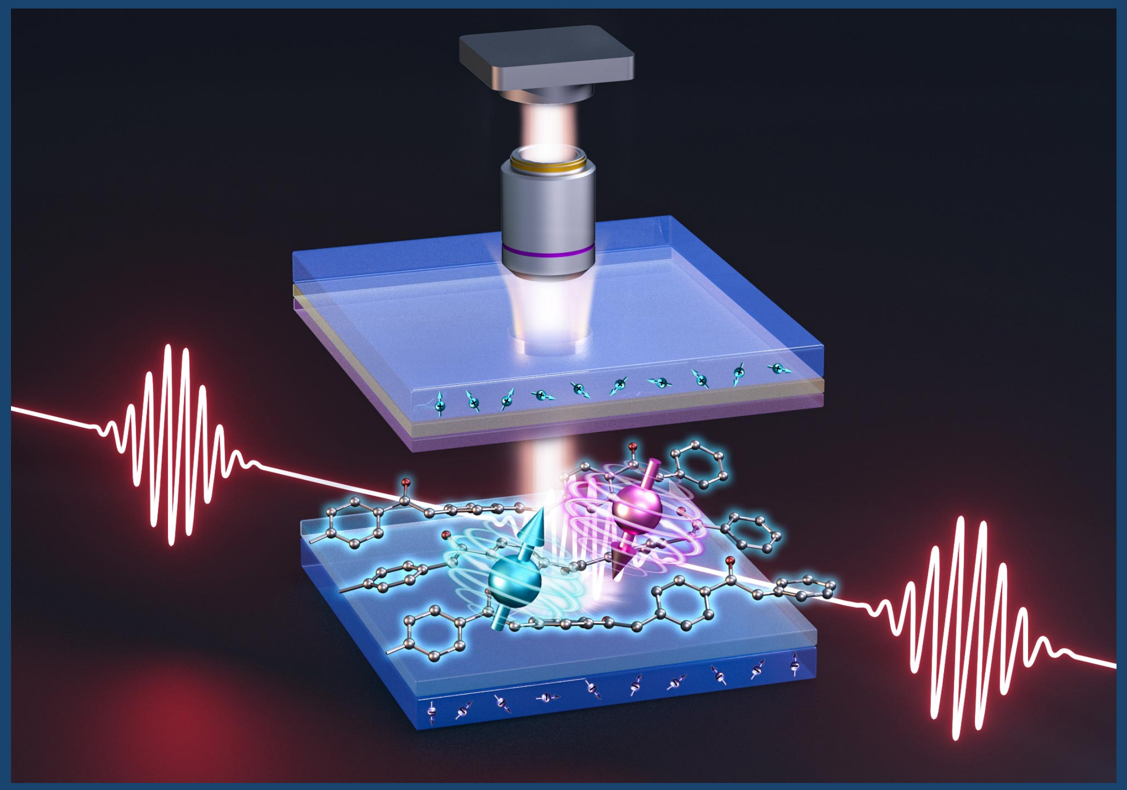
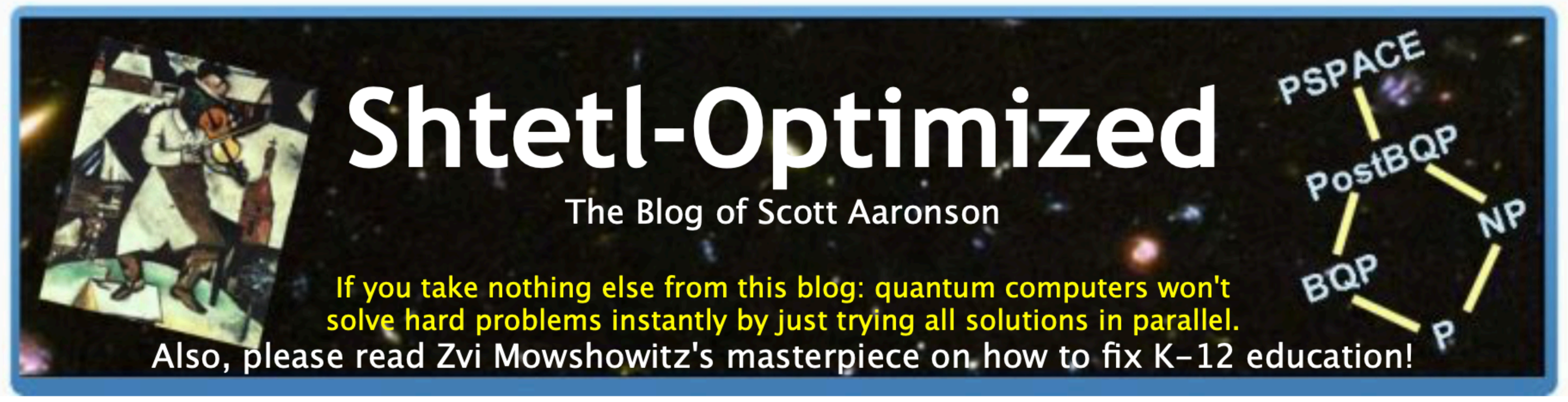


Quantum sensing

Quantum Hermite Transform



« BusyBeaver(6) is really quite large

ChatGPT and the Meaning of Life: Guest Post by Harvey Lederman »

Quantum Complexity Theory Student Project Showcase #5 (2025 Edition)!

Sorry for the long blog-hiatus! I was completely occupied for weeks, teaching an intensive course on theoretical computer science to 11-year-olds (!), at a math camp in St. Louis that was also attended by my 8-year-old son. Soon I'll accompany my 12-year-old daughter to a science camp in Connecticut, where I'll also give lectures.

There's a great deal to say about these experiences, but for now: it's been *utterly transformative and life-affirming* to spend my days in teaching precocious, enthusiastic, non-jaded children the material I love in the real world, rather than (let's say) in scrolling through depressing world news and social media posts and emails from hateful trolls on my phone. It's made me feel 25 years younger (well, at least until I need to walk up a flight of stairs). It's made me want to go back to actual research and teaching, which besides family and friends have been the main sources of joy in my life.

So on that note, and without further ado: I hereby present the latest **Quantum Complexity Theory Student Project Showcase!** As the name suggests, this is where I share a selection of the best research projects, from the students who took my graduate class on Quantum Complexity Theory at UT Austin this past spring.

Quantum Hermite Transform and Gaussian Goldreich-Levin

Vishnu Iyer — vishnu.iyer@utexas.edu
Siddhartha Jain — sidjain@utexas.edu
The University of Texas at Austin

July 30, 2025

Abstract

The representation of a function $f : \mathbb{R}^n \to \mathbb{R}$ as a linear combination of Hermite polynomials can be seen as the Gaussian analogue of the Fourier expansion for Boolean functions. Strengthening this analogy, we show that an approximate Hermite transform can be implemented efficiently on quantum computers given black-box access to f. This implies that the Gaussian analogue of the Goldreich-Levin learning problem can be solved on quantum computers with query complexity independent of n. With these tools, we give examples of provable quantum advantage via Hermite sampling.

1 Introduction

Most quantum complexity literature focuses on problems with discrete inputs, but it can pay off to study continuous variable inputs. An early example of this is the observation by Jordan [Jor05] that the Bernstein-Vazirani problem [BV97] looks like computing a gradient. Jordan generalized the Bernstein-Vazirani algorithm to a function with inputs in $\mathbb{R}^n$ to give a single quantum query numerical gradient estimation algorithm.

Efficient Quantum Hermite Transform

arXiv > quant-ph > arXiv:2510.04929

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Quantum Physics

[Submitted on 6 Oct 2025]

Efficient Quantum Hermite Transform

Siddhartha Jain, Vishnu Iyer, Rolando D. Somma, Ning Bao, Stephen P. Jordan

We present a new primitive for quantum algorithms that implements a discrete Hermite transform efficiently, in time that depends logarithmically in both the dimension and the inverse of the allowable error. This transform, which maps basis states to states whose amplitudes are proportional to the Hermite functions, can be interpreted as the Gaussian analogue of the Fourier transform. Our algorithm is based on a method to exponentially fast forward the evolution of the quantum harmonic oscillator, which significantly improves over prior art. We apply this Hermite transform to give examples of provable quantum query advantage in property testing and learning. In particular, we show how to efficiently test the property of being close to a low- degree in the Hermite basis when inputs are sampled from the Gaussian distribution, and how to solve a Gaussian analogue of the Goldreich-Levin learning task efficiently. We also comment on other potential uses of this transform to simulating time dynamics of quantum systems in the continuum.

To do this, we need to show how to simulate the quantum harmonic oscillator (a **spring**!) upto energy E in time $O(\log^2 E)$.

Previous best was exponentially worse.

The Feynman/Manin program

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have my own things to say and to talk about and there's no implication that anybody needs to talk about the same thing or anything like it. So what I want to talk about is what Mike Dertouzos suggested that nobody would talk about. I want to talk about the problem of simulating physics with computers and I mean that in a specific way which I am going to explain. The reason for doing this is something that I learned about from Ed Fredkin, and my entire interest in the subject has been inspired by him. It has to do with learning something about the possibilities of computers, and also something about possibilities in physics. If we suppose that we know all

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What else is waiting to be found?

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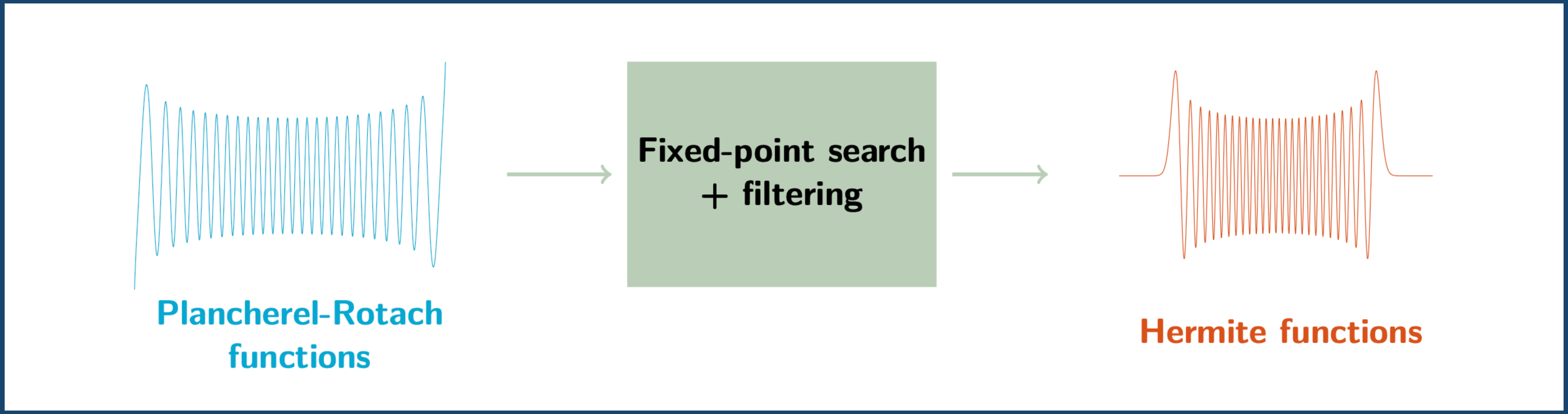

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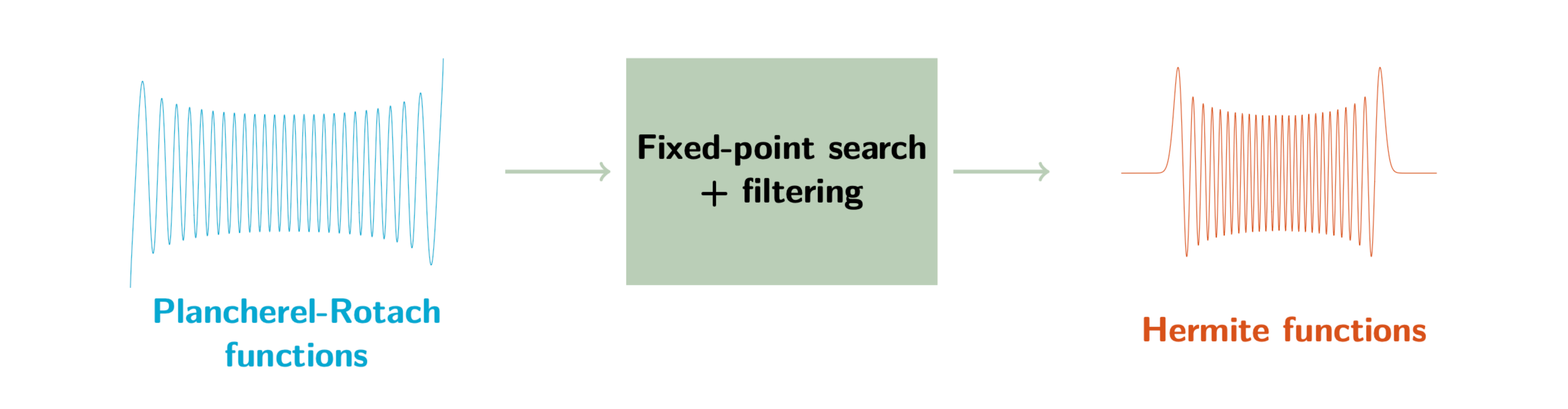

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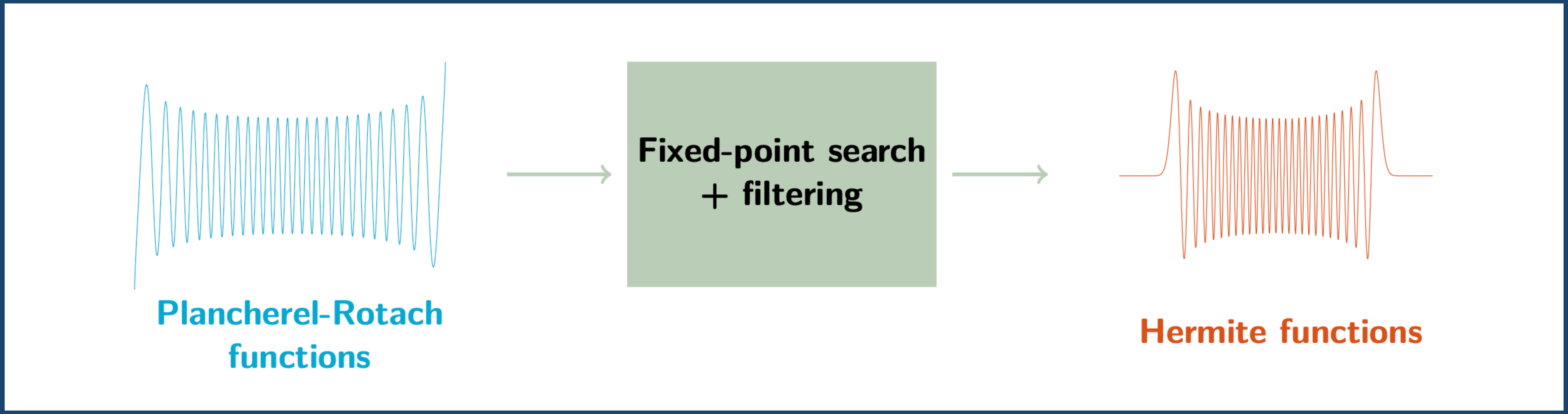

- Start with functions with $\Theta(1)$ overlap (Plancherel-Rotach)
- Use fixed-point search with optimal queries to converge to eigenstate of quantum harmonic oscillator (uses fast-forwarding!)

Fast-forwarding QHO in the continuum

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Naive hope: $e^{x^2+p^2} = e^{x^2} e^{p^2}$

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$$Z = X + Y + \frac{1}{2}[X, Y] + \frac{1}{12}[X, [X, Y]] + \frac{1}{12}[Y, [Y, X]] + \cdots$$

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What about discretization?

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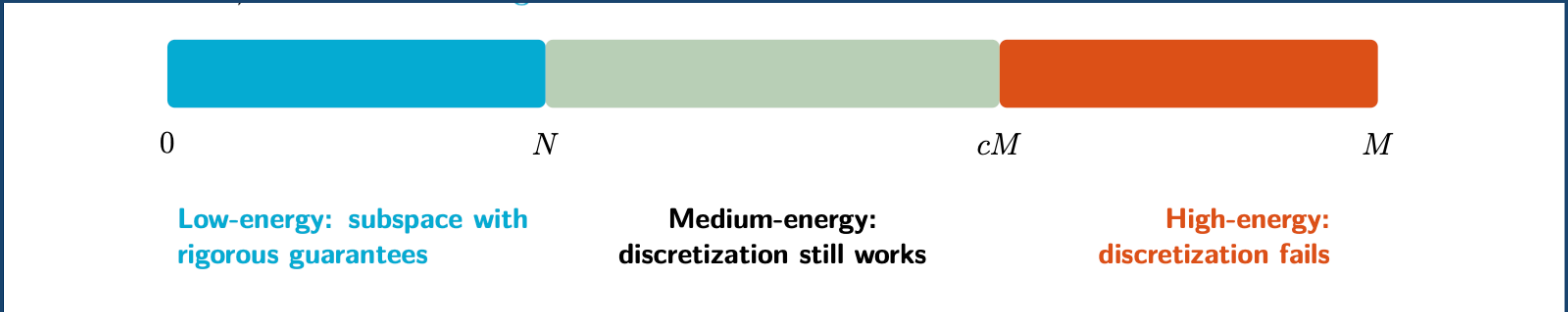
Then we want that,

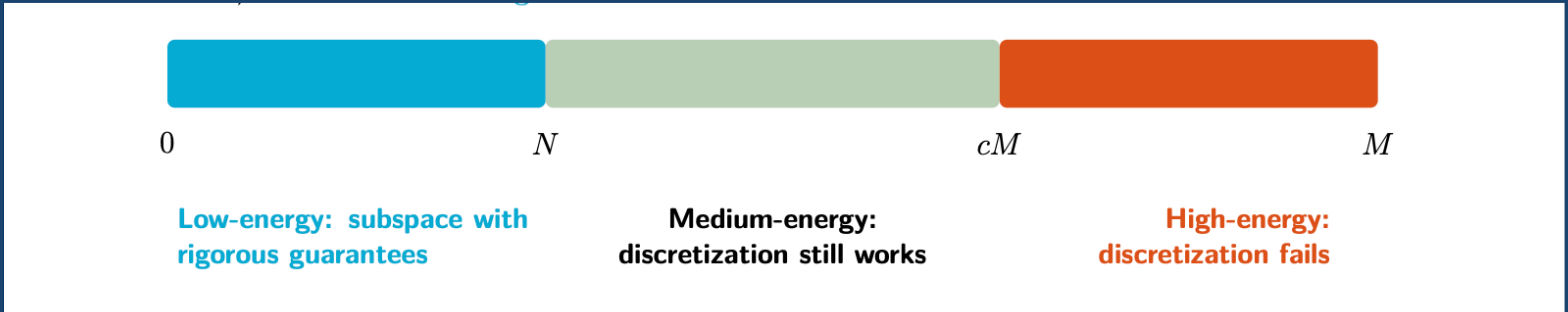
$$\|\Pi_N\left(e^{i\overline{H}t} - \widetilde{U}(\overline{x},\overline{p})\right)\Pi_N\| \leq \exp(-\Omega(N))$$

Fast-forwarding QHO



$$N = O(M/\log M)$$

Fast-forwarding QHO



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Tricks required:

- Leakage bounds, example
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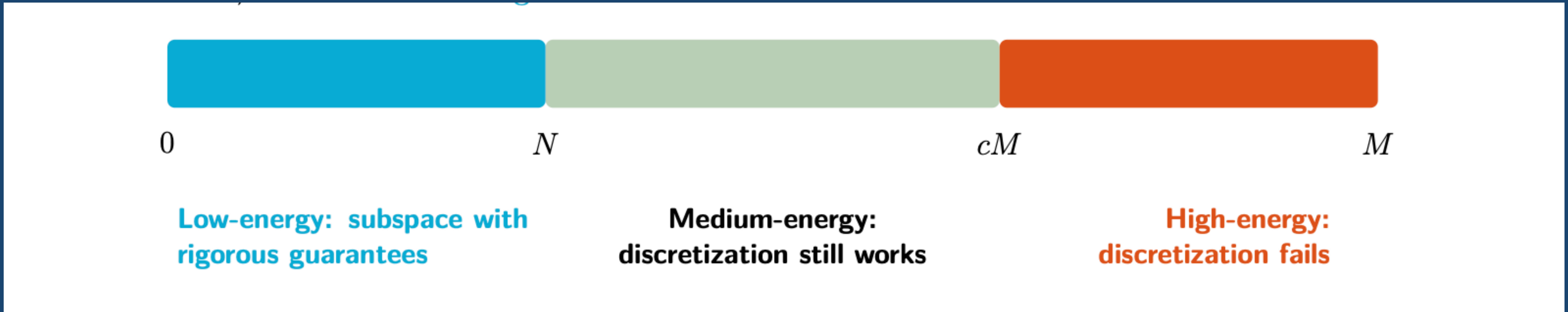

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Tricks required:

- Leakage bounds, example $\|(I - \Pi_{N'})\bar{x}^a \Pi_N\| \leq \exp(-\Omega(M))$
- Operator norm bound in low-energy subspace, example $\|\Pi_N \bar{x}^{2t} \Pi_N\| \leq O(N^t)$

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- Applications to differential equations/quantum chemistry?

Thanks for your attention!

arXiv:2510.04929