# Consumable Data via Quantum Communication

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Quantum AI

“The economics of data is a new but rapidly growing field.”

Jones & Tonetti, *Nonrivalry and the Economics of Data*

# Nonrivalry of classical data

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Can be copied at essentially zero cost

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Concerns

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### Hope

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As an example, in ML

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Destructive measurement in quantum mechanics to the rescue!

# Nonrivalry of classical data

## Can be copied at essentially zero cost

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As an example, in ML

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### Hope

Destructive measurement in quantum mechanics to the rescue!

# Our model: Communication Complexity

# Communication Complexity

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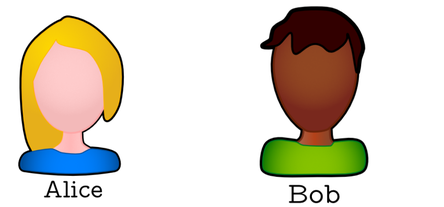
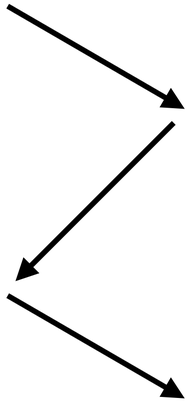


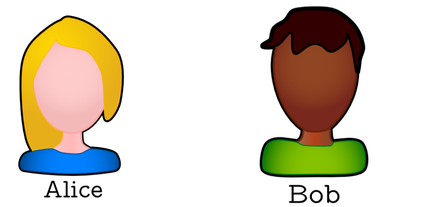




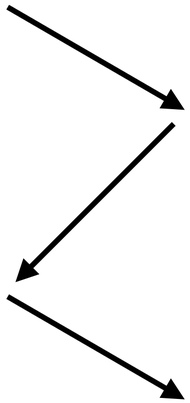



z: (x,y,z) is a solution

# Communication Complexity







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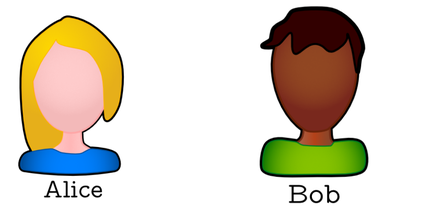
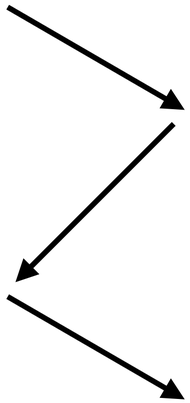
Expressive

# Communication Complexity







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## Expressive

Can simulate models like query complexity, circuits, property testing, streaming and more

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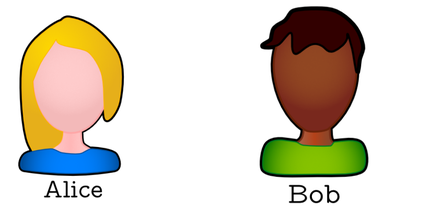



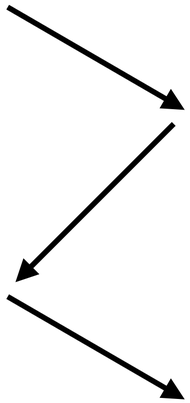

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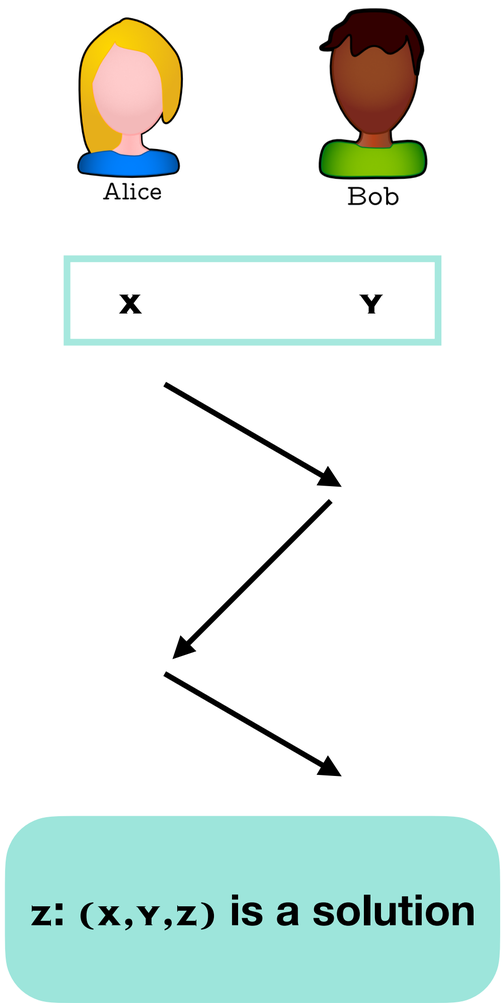


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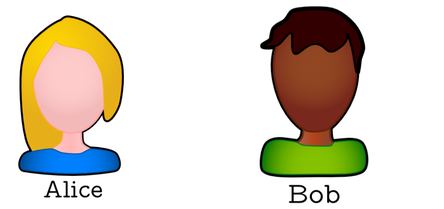
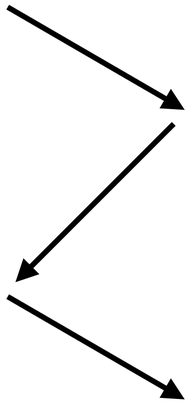
Can simulate models like query complexity, circuits, property testing, streaming and more

## Tractable

# Communication Complexity







z: (x,y,z) is a solution

## Expressive

Can simulate models like query complexity, circuits, property testing, streaming and more

## Tractable

Can prove unconditional lower bounds for problems we care about

“Communication is everything,
everything is communication.”

Folklore

# An example market

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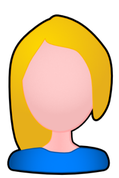
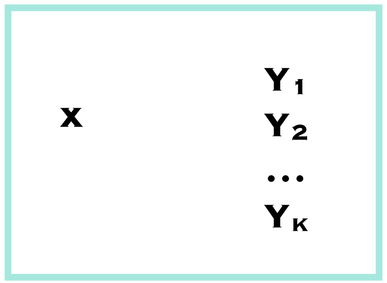
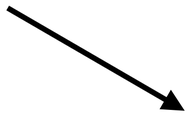


Alice

Bob

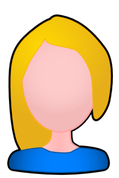


**Solutions for**
$(x,y_1)$, ... $(x,y_K)$

# An example market

Data



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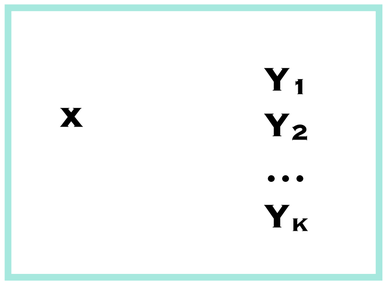


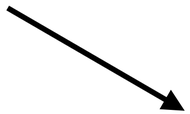

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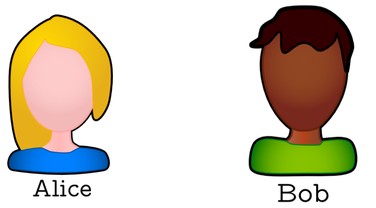





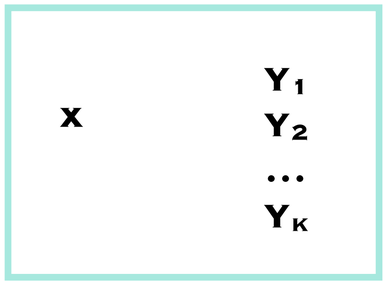


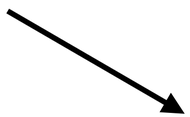

**Solutions for**
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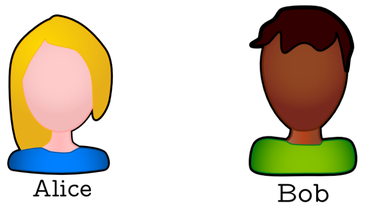
## Data

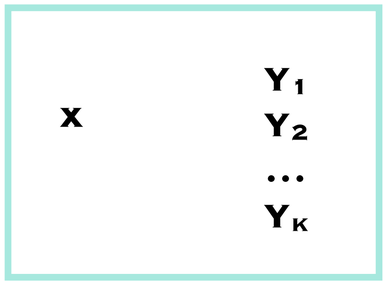
Alice holds some data $x$ which is useful for training models

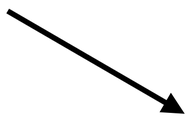
# An example market









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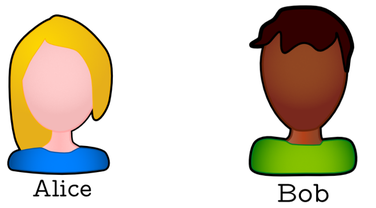


| x | $Y_1$<br>$Y_2$<br>…<br>$Y_K$ |
|---|---|

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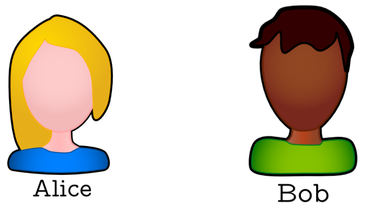


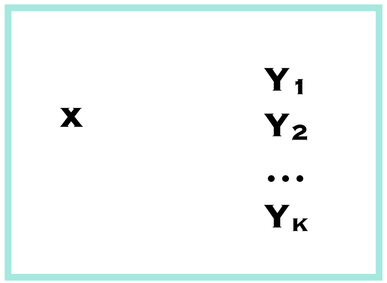


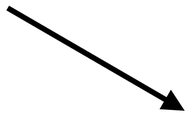

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Bob has $K$ machine learning models which he wants to train using Alice's data

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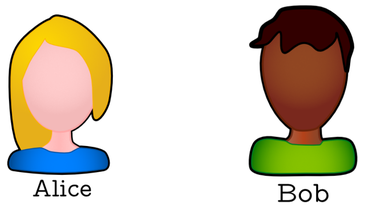


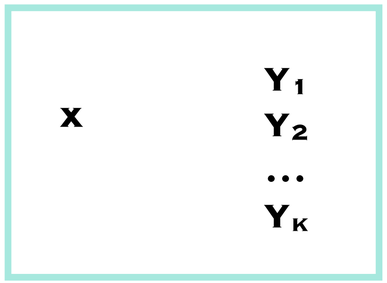


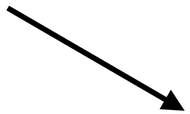

**Solutions for**
$(x,Y_1)$, ... $(x,Y_K)$

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## Buyer

Bob has $K$ machine learning models which he wants to train using Alice's data

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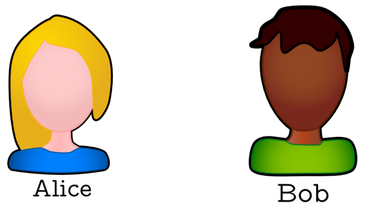


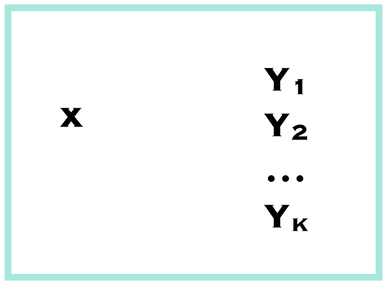


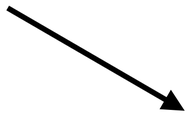

**Solutions for**
$(x,y_1)$, ... $(x,y_\kappa)$

## Data

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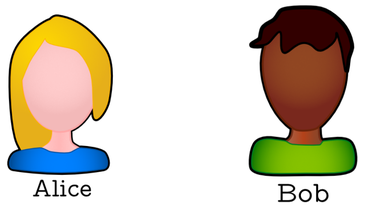
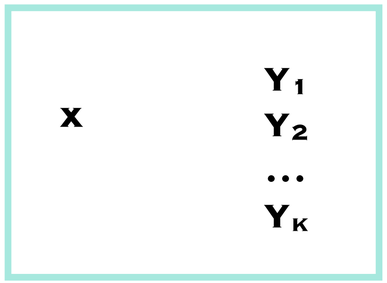
## Buyer

Bob has $\kappa$ machine learning models which he wants to train using Alice's data

## Communication

# An example market




**Solutions for**
$(x,Y_1)$, ... $(x,Y_K)$

## Data

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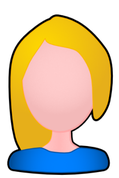
## Buyer

Bob has $K$ machine learning models which he wants to train using Alice's data

## Communication

On payment, Alice sends a copy of her data to Bob

# Consumable Data



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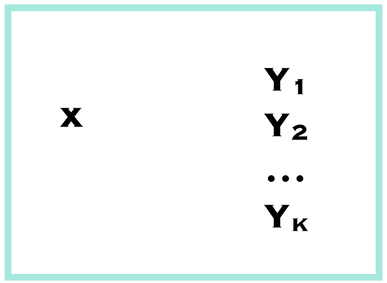


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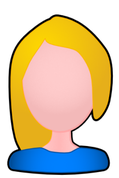
# Consumable Data

## Asymmetric Direct Sum for One-way Communication



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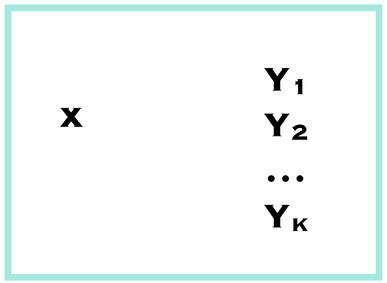


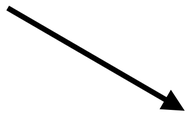

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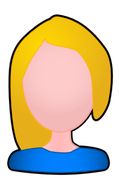

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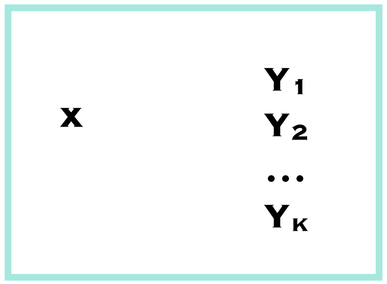


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Defintion

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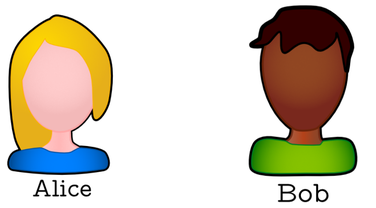
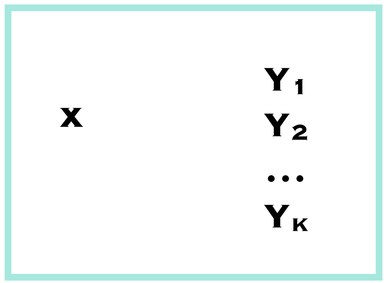
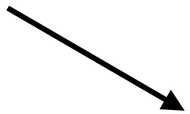








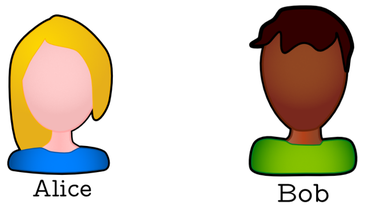
**Solutions for**
$(x,Y_1)$, ... $(x,Y_k)$

### Defintion

$R$ is a consumable data problem if $R^k$ requires $poly(k)\ CC(R)$ bits to be communicated

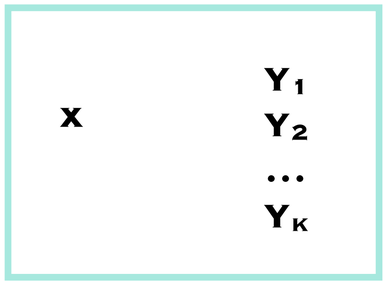
# Consumable Data

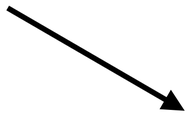
## Asymmetric Direct Sum for One-way Communication









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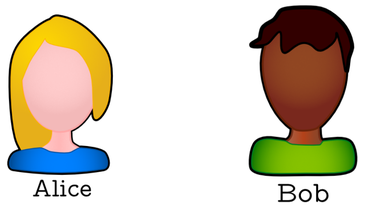





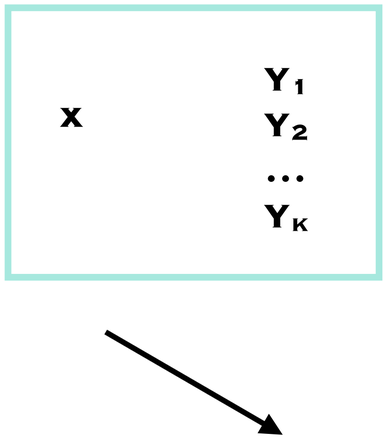


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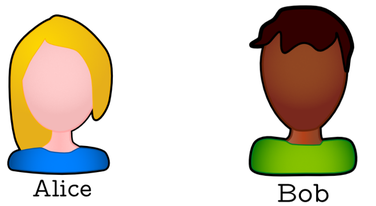
### Strong version

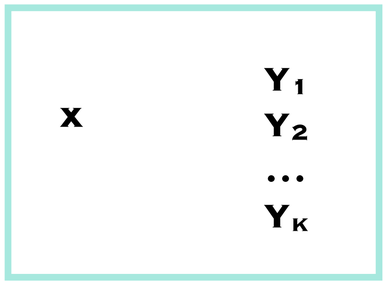
# Consumable Data

## Asymmetric Direct Sum for One-way Communication










**Solutions for**
$(x,Y_1)$, ... $(x,Y_k)$

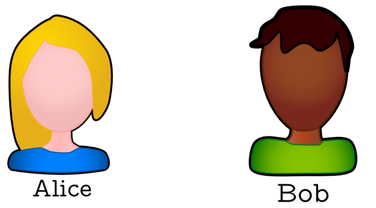
### Defintion

R is a consumable data problem if $R^k$ requires poly(k) CC(R) bits to be communicated

### Strong version

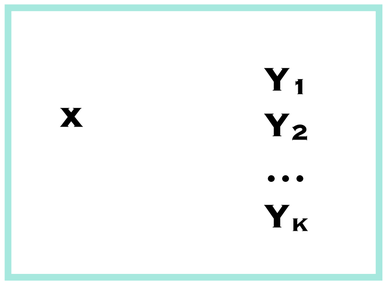
R is a strongly consumable data problem if the same lower bound holds when Bob wants to solve any 2/3 fraction of instances

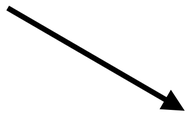
# Our results









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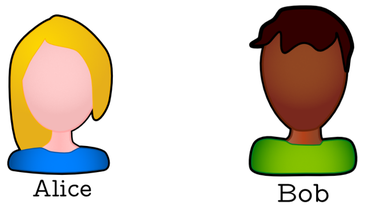





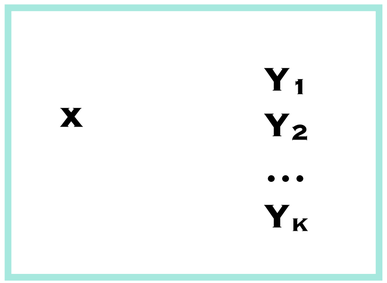


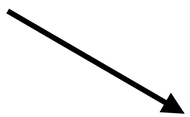

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Proof-of-concept examples

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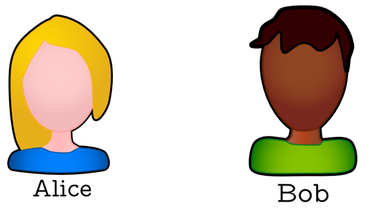





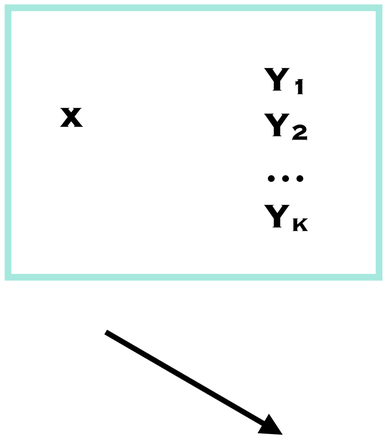


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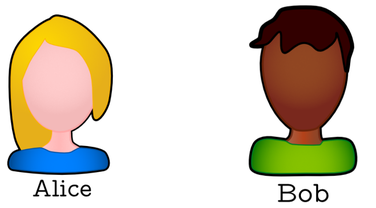
## Proof-of-concept examples

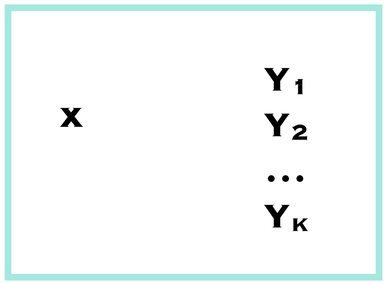
1. Linear Regression Sampling

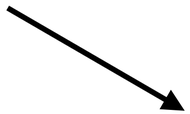
# Our results









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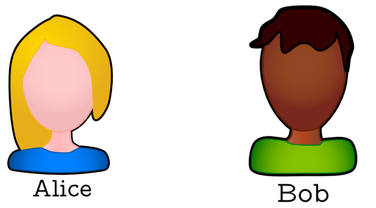





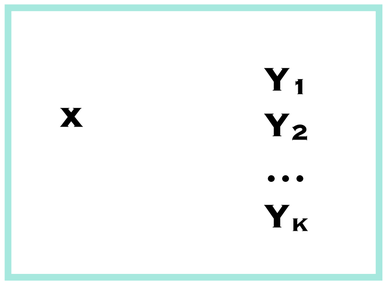


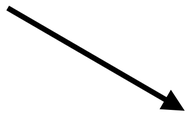

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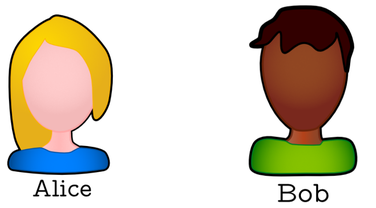





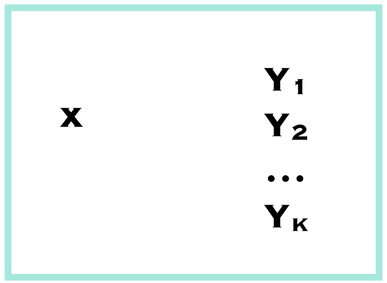


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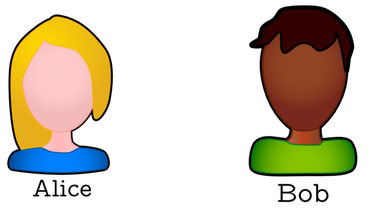





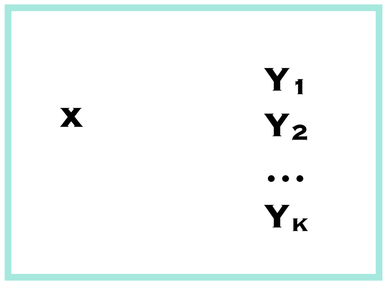


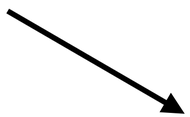

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1. Linear Regression Sampling
2. Hidden Matching

## Impossibility results

Decision problems

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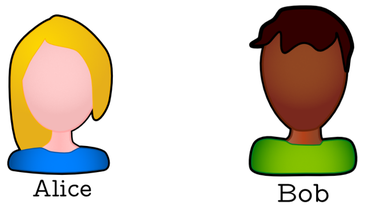





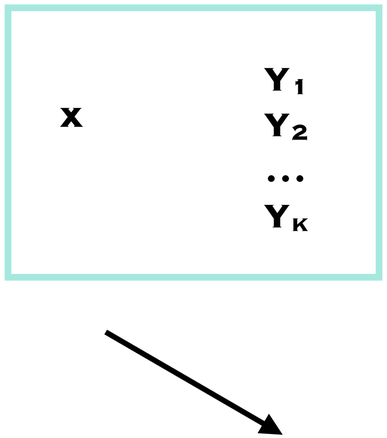


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## Proof-of-concept examples

1. Linear Regression Sampling
2. Hidden Matching

Linear

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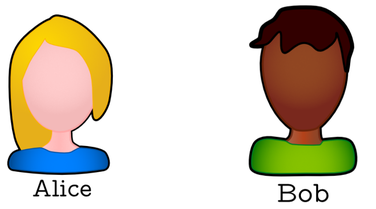





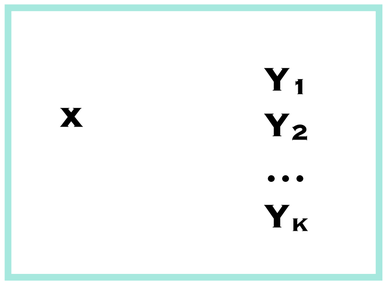


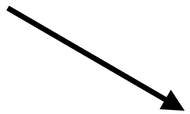

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1. Linear Regression Sampling
2. Hidden Matching

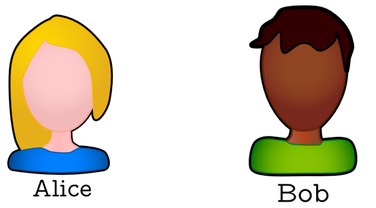
Linear

Square-root

## Impossibility results

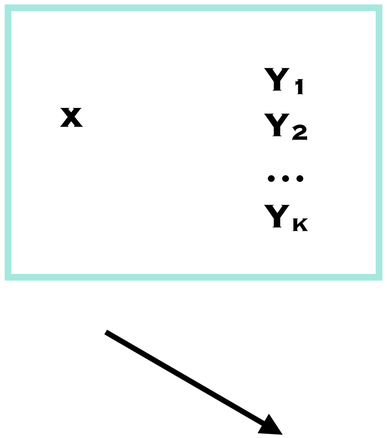
Decision problems

# Impossibility for decision problems







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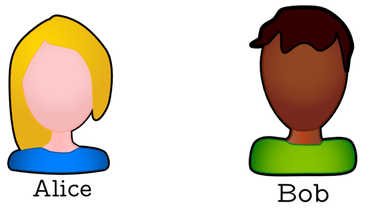





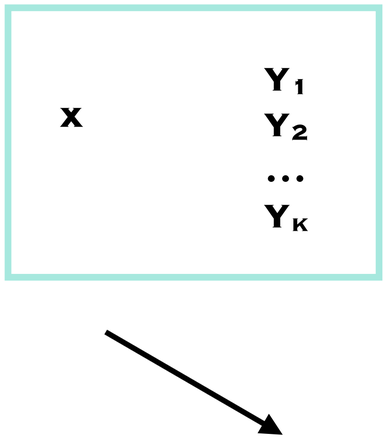


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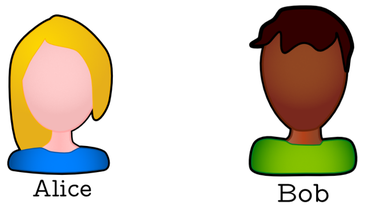





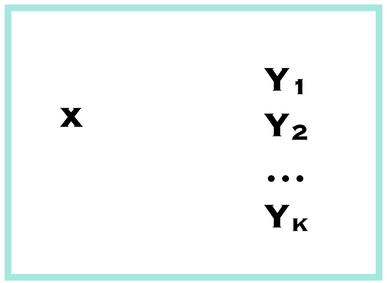


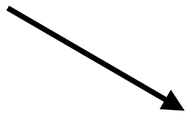

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Introduced by Aaronson (2017), allows us to estimate the values of $K$ two-outcome observables applied to a $N$ qubit state using only $\text{POLYLOG}(N,K)$ samples

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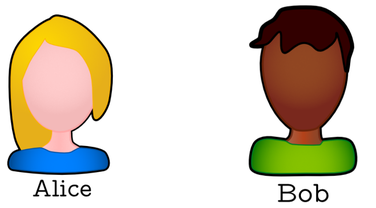





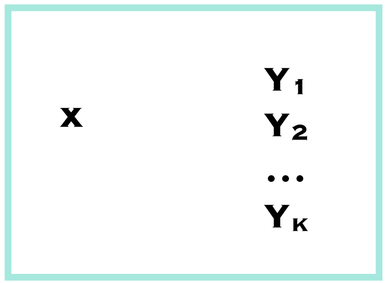


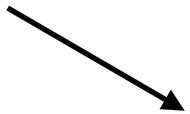

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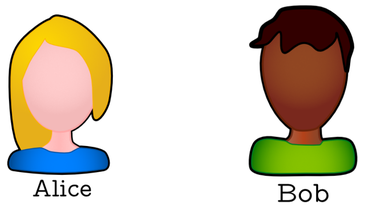





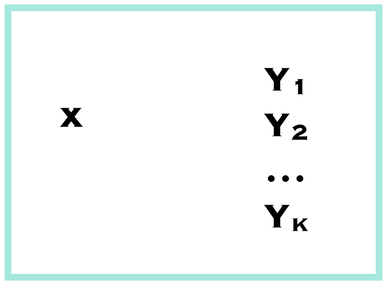


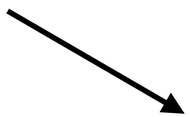

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This gives rise to an easy communication protocol

# Application: A fair data auction

# Posted price data auction

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$$V_A = PB$$

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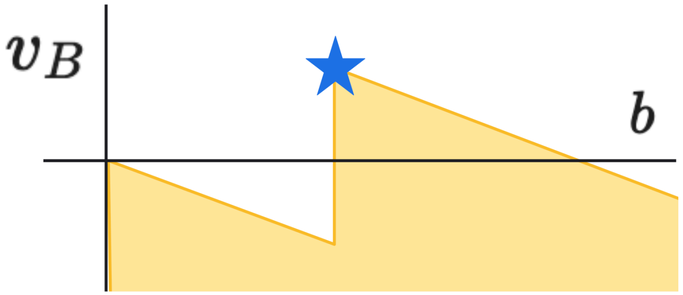


Classical

- $B^*$ is independent of $K$
- Alice doesn't know $K$
- Alice's payoff is $O(1)$

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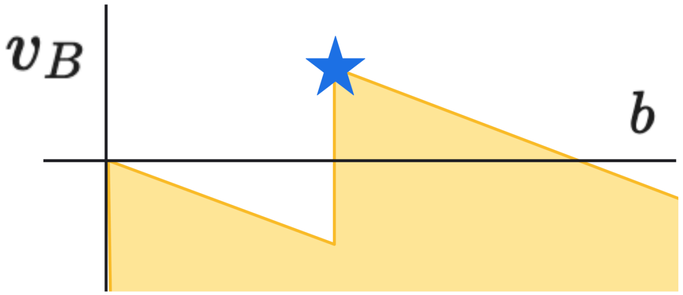


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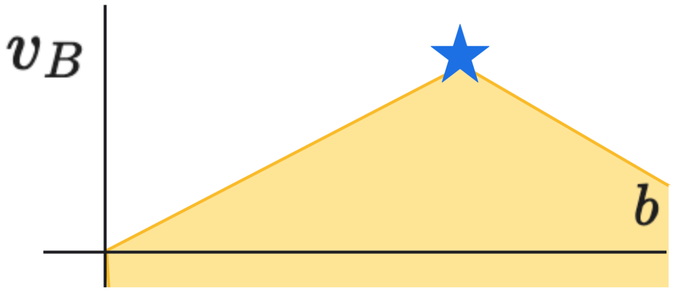


## Quantum

- $B^*$ scales linearly with $K$
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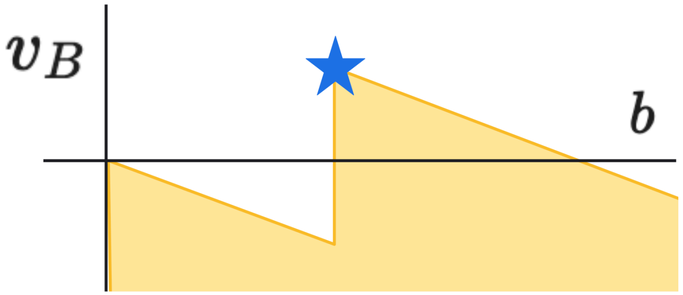


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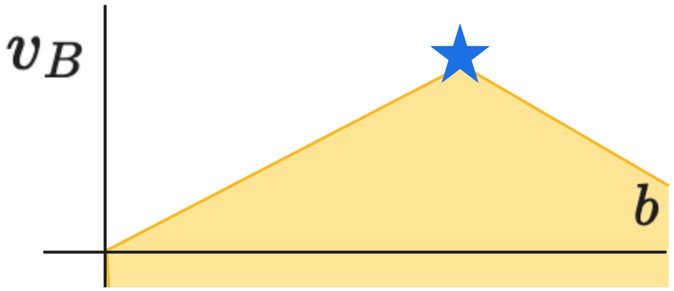


## Quantum

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## Takeaway

When using quantum communication, Alice's payoff is proportional to the number of times $x$ is used to generate solutions by Bob.

# Future work

# Open problems

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Proof needs to avoid classical upper bound when $\kappa > \sqrt{N}$

# Thanks for listening!

Au revoir